Malaysian Journal of Mathematical Sciences 9(3): 397–407 (2015)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

New Types of Functions and Separation Axiom by γ-Open Sets in Fuzzy Bitopological Spaces

Arnab Paul *[1] and Baby Bhattacharya [2]

[1,2] *National Institute of Technology Agartala, India*

*E-mail: * mrarnabpaul87@gmail.com*
**Corresponding author*

ABSTRACT

The main aim of this paper is to introduce a new class of fuzzy irresolute functions called (i,j)-completely fuzzy γ-irresolute function in fuzzy bitopological space and to study some basic properties of the newly defined function.Then we define a weaker form of (i,j)-completely fuzzy γ-irresolute function termed as(i,j)-fuzzy γ-irresolute function.Finally we study a new type of fuzzy bitopological space namely (i,j)-fuzzy γ-normal space.We obtain an additional condition under which (i,j)- fuzzy γ^*-closed surjection function preserves (i,j) -fuzzy γ-normality.

Keywords: (i,j)-fuzzy γ-open set,(i,j)-fuzzy γ–irresolute function,(i,j)-completely fuzzy γ-irresolute function.

1. Introduction

The notion of bitopological spaces was introduced and investigated at the initial stage by Kelly (1963) and thereafter a large number of papers are done to generalize the topological ideas to bitopological spaces. Functions and in fact irresolute functions offer new path towards analysis. Mukherjee (1985) defined the irresolute mappings in bitopological spaces. For more details on alternative forms of irresolute functions in topology and fuzzy topological spaces see for example, Chitharanjan and Missier (2012), Ekici and Jafari (2008), Khedr and Al-Saadi (2012), Park and J. (1998), Rajesh (2011), Thakur and Malviya (1995) and Zorlutuna (2008). The notion of fuzzy set was introduced by Zadeh (1965). The potential of the introduced notion was realised by the researchers and has been applied with success for investigations in all the branches of science and technology.

Then Kandil (1989) introduced the idea of fuzzy bitopological spaces as an extension of fuzzy topological spaces and a generalization of bitopological spaces. Bhaumik and Pal (1993) studied and introduced the concepts of completely and weakly irresolute function in fuzzy topological space. Nour (1995) has studied some separation axiom in bitopological space following the pairwise separation axiom of Kelly (1963). Subsequently Thakur and Malviya (1995) introduced the ideas of pairwise fuzzy irresolute mappings. Presently Tripathy and Debnath (2013) have introduce γ-open set in fuzzy bitopological space and shown that every (1, 2)-fuzzy pre open set may not be a (1, 2)-fuzzy γ-open set. Tripathy and Sharma (2013) introduced pairwise strongly b-open and pairwise strongly b-closed functions in bitopological spaces and studied the connection with pairwise strongly b-normal and pairwise b-separated spaces. Also similar notions have been introduced in the area of L-fuzzy topology by Aygun and Kudri (1999) and Mukherjee and Bhattacharya (2003). Naschie (1998, 2000) has shown that the notion of fuzzy topology may be relevant to quantum particle physics in connections with string theory and ϵ^{∞} theory.

The aim of this paper is to introduce a new type of irresolute function namely(i,j)-completely fuzzy γ-irresolute function in fuzzy bitopological space. Some basic properties and characterization of this newly defined function are mentioned.In this paper we tend to outline the ideas of (i,j)-fuzzy γ-irresolute function,(i,j)-fuzzy γ^*-closed mapping,(i,j)-fuzzy γ-normal spaces and try to investigate the properties of this newly defined ideas. Also our special concentration is to show that every τ_1-fuzzy open set may not be a (1,2)-fuzzy γ-open set and every τ_2-fuzzy open set may not be a (2,1)-fuzzy γ-open set.

Throughout this paper X and Y denote the fuzzy bitopological spaces (X,τ_1,τ_2) and (Y,σ_1,σ_2)respectively. A fuzzy subset μ of a fuzzy bitopological space (X,τ_1,τ_2) is said to be:

i. (i,j)-fuzzy preopen(citeKumar1994) if $\mu \leq$i-int(j-cl(μ)), where i$\neq$j, i,j $=$1,2.

ii. (i,j)-fuzzy γ-open(Tripathy2013) if $\mu \cap \beta$is(i, j)-fuzzy preopen for every (i, j)-fuzzy preopen set β in$X$, where is $\neq$j, i,j $=$1,2.

iii. (i,j)-fuzzy regular open (Khedr and Al-Saadi (2005)) if $\mu=$i-int(j-cl(μ)),where i$\neq$j, i,j $=$1,2.

iv. A function$f$:$X \longrightarrow Y$ from a fuzzy topological space $(X,\tau)$to another fuzzy topological space $(Y,\sigma)$is said to be fuzzy completely irresolute function (Bhaumik and Pal (1993)) iff $f^{-1}(\alpha)$ is fuzzy regular open subsets of X for every fuzzy semi-open subset α in Y.

For a function f:$X \longrightarrow Y$,a function g:$X \longrightarrow X$XY, defined by $g(x)=(x,f(x))$for each $x \in X$ is called the graph function of f.

The subset $\{(x,f(x)):x \in X\}$ is called the graph of f and is denoted by *G(f)*.The family of all (i,j)-fuzzy regular open (resp. (i,j)-fuzzyγ-open) sets of a fuzzy bitopological space $(X,\tau_1,\tau_2)$is denoted by(i,j)-FR$O(X)$(resp.(i,j)F$\gamma O(X)$).

2. Properties of (i,j)-Completely Fuzzy Y γ-Irresolute Functions

In this section we define (i,j)-completely fuzzy γ-irresolute function and study some of their basic properties and characterizations.

Definition 2.1. *A function f:$(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$is called (i,j)-completely fuzzy γ-irresolute function if the inverse image of each(i,j)-fuzzy γ-open set in Y is (i,j)-fuzzy regular open set in X.*

Remark 2.1. *In a fuzzy bitopological space an (i,j) - fuzzy γ-continuous function may not be a (i,j)-completely fuzzy γ-irresolute function as seen in the following example:*

Example 2.1. *Let $X=\{x,y\}$ and $Y=\{a,b\}$.Consider the fuzzy set on X are $\overline{0}$,$\overline{1}$,A,B,where A$=\{$(x,0.4),(y,0.8)$\}$ and B$=\{$(x,0.2),(y,0.3)$\}$.Also consider the fuzzy sets in Y are $\overline{0}$,$\overline{1}$,C, where C$=\{$(a,0.5),(b,0.5)$\}$.*

Now let τ_1=$\{\overline{0},\overline{1},A\}$,$\tau_2$=$\{\overline{0},\overline{1},B\}$,$\sigma_1$=$\{\overline{0},\overline{1}\}$ and σ_2=$\{\overline{0},\overline{1},C\}$.

We get (i,j)-FγO(Y)=$\{\overline{0},\overline{1},\{(a,\alpha),(b,\beta)\}\}$ where $\alpha>0.95$, and $\beta>0.95$ and also we get (i,j)-FRO(X)=$\{\overline{0},\overline{1}\}$.

We define a function f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ such that $f(x)=a$ and $f(y)=b$.Thus the inverse image of each σ_1-fuzzy open set in Y is(i,j)-fuzzy γ-open set in X . But under this function the inverse image of(i, j)-fuzzy γ-open set $\{(a,0.96),(b,0.96)\}$ in Y is $\{(x,0.96),(y,0.96)\}$ which is not a (i,j)-fuzzy regular open set in X. Hence f is not a (i,j)-completely fuzzy γ-irresolute function.

Theorem 2.1. *For a function f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ the following properties are equivalent:*

i. *f is (i,j)-completely fuzzy γ-irresolute function.*

ii. *$f^{-1}(V)\subset$(i,j)-FRint(f^{-1}(i,j)-Fγcl(V))$\forall V\in$(i,j)-FγO(Y) and*

iii. *(i,j)-FRcl($f^{-1}(V)$)$\subset f^{-1}$((i,j)-Fγcl(V)).*

Proof. (i)$\rightarrow$(ii) Let $V\in$(i,j)-FγOY and $x\in f^{-1}(V)$.Thus $f(x)\in V$. Now since f is (i,j)-completely fuzzy γ-irresolute function, so there exist a $U\in$FRO($\mathbf{X}$,x) such that $f(U)\subset V\Rightarrow f(U)\subset$(i,j)-F$\gamma$cl($V$). Therefore $x\in U\subset f^{-1}$((i,j)-Fγcl(V)). This shows that $f^{-1}(V)\subset$(i,j)-FRint(f^{-1}(i,j)-Fγcl(V)).

(ii)$\rightarrow$(iii) Let $V\in$(i,j)-FγO(Y). Suppose $x\notin f^{-1}$((i,j)-Fγcl(V)). Thus there exist a (i,j)-fuzzy γ-open set W in Y containing $f(x)$ such that $W\cap V=\overline{0}$.

Hence (i,j)-Fγcl(W)$\cap V=\overline{0}$.
$\Rightarrow$ f^{-1}((i,j)-Fγcl(W))$\cap f^{-1}(V)=\overline{0}$. Now since $x\in f^{-1}(W)$then by (ii) we get $x\in$(i,j)-FRint(f^{-1}(i,j)-Fγcl(W)). Thus there exist a $U\in$(i,j)-FRO(X,x) such that $U\subset f^{-1}$(i,j)-Fγcl(W)). Thus we get $U\cap f^{-1}(V)=\overline{0}$ $\Rightarrow x\notin$(i,j)-FRcl($f^{-1}(V)$).

Hence the proof. (iii)$\rightarrow$(i) Suppose $x\in X$ and $V\in$(i,j)-FγO(Y,$f(x)$). Thus $f(x)\notin$(i,j)-Fγcl($Y-V$). Therefore $x\notin f^{-1}$((i,j)-Fγcl($Y-V$)) and by (iii) we have $x\notin$(i,j)-FRcl($f^{-1}(Y-V)$). Therefore there exist $U\in$(i,j)-FRO(X,x) such that $U\cap f^{-1}(Y-V)=\overline{0}$.
$\Rightarrow$ $U\subset f^{-1}(V)$.

This shows that f is (i,j)-completely fuzzy γ-irresolute function □

Definition 2.2. *A function f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$is called (i,j)-fuzzy γ-irresolute function if the inverse image of each (i,j)-fuzzy γ-open set in Y is (i,j)-fuzzy γ-open set in X.*

Proposition 2.1. *In a fuzzy bitopological space every (i,j)-completely fuzzy γ-irresolute function is a (i,j)-fuzzy γ-irresolute function.*

Remark 2.2. *Converse of the above proposition may not be true as seen in the following example:*

Example 2.2. *Let X=$\{x,y\}$ and Y=$\{a,b\}$. Consider the fuzzy set on X are $\overline{0}$,$\overline{1}$,A and Bwhere A=$\{$(x,0.4),(y,0.3)$\}$ and B=$\{$(x,0.2),(y,0.3)$\}$.Also consider the fuzzy sets in Y are $\overline{0}$,$\overline{1}$,C,D where C=$\{$(a,0.2),(b,0.3)$\}$ and D=$\{$(a,0.1),(b,0.3)$\}$. Now let $\tau_1=\{\overline{0},\overline{1},A\}$,$\tau_2=\{\overline{0},\overline{1},B\}$,$\sigma_1=\{\overline{0},\overline{1},C\}$ and $\sigma_2=\{\overline{0},\overline{1},D\}$.*

$$\textit{We get(i,j)-F}\gamma O(X) = \begin{cases} \{(x,\alpha),(y,\beta)\}, & \textit{for } 0 \le \alpha \le 0.4, 0 \le \beta \le 0.3 \\ \{(x,\alpha),(y,\beta)\}, & \textit{for } \alpha > 0.8 \ \ \textit{and} \ \ \beta > 0.7 \end{cases}$$

$$\textit{We get(i,j)-F}\gamma O(Y) = \begin{cases} \{(a,\alpha),(b,\beta)\}, & \textit{for } 0 \le \alpha \le 0.2, 0 \le \beta \le 0.3 \\ \{(a,\alpha),(b,\beta)\}, & \textit{for } \alpha > 0.9 \ \ \textit{and} \ \ \beta > 0.7 \end{cases}$$

Now we define a function f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ such that f(x)=a and f(y)=b. Here inverse image of every (i,j)-fuzzy γ-open set in Yis (i,j)-fuzzy γ-open set in X. But f^{-1}(D)=$\{$(x,0.1),(y,0.3)$\}$ which is not a (i,j)-fuzzy regular open set in X although D is (i,j)-fuzzy γ-open set in Y.

Now we find an additional condition and using it we show that the converse of the above **Proposition 2.1** holds true in the following theorem:

Theorem 2.2. *If f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is a(i,j)-fuzzy γ-irresolute function and every subset is a (i,j)-fuzzy pre-closed set in X then f is (i,j)-completely fuzzy γ-irresolute function.*

Proof. Let A be a (i,j)-fuzzy γ-open set in in Y.Now since f is (i,j)-fuzzy γ-irresolute function ,therefore $f^{-1}(A)$is (i,j)-fuzzy γ-open set in X.This implies $f^{-1}(A)$is (i,j)-fuzzy pre-open set in X. Again from the hypothesis $f^{-1}(A)$is (i,j)-fuzzy pre-closed set in X. Thus $f^{-1}(A)$ is (i,j)-fuzzy regular open set in X. Hence f is a (i,j)-completely fuzzy γ-irresolute function. □

Theorem 2.3. *For the function f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ the following statements are equivalent:*

i. *f is (i,j)-completely fuzzy γ-irresolute and*

ii. *f^{-1}(U) is (i,j)-fuzzy regular closed set in X for every (i.j)-fuzzy γ-closed set U of Y.*

Proof. The proof is straight forward. □

Theorem 2.4. *If a function f:(X,τ_1,τ_2)→(Y,σ_1,σ_2) is (i,j)-completely fuzzy γ-irresolute and A is τ_i-fuzzy open set in X, then the restriction function $f|_A$:A→Y is (i,j)-completely fuzzy γ-irresolute.*

Proof. Let U be any (i,j)-fuzzy γ-open set in Y.Since f is a (i,j)-completely fuzzy γ-irresolute, so $f^{-1}(U)$ is (i,j)-fuzzy regular open set in X. Now since $(f^{-1}(U)\cap A)$is (i,j)-fuzzy regular opens set in the subspace $(A, (\tau_i\tau_j)_A)$ so $f^{-1}(U)\cap A{=}(f|_A)^{-1}(U)$. Hence $f|_A$ is (i,j)-completely fuzzy γ-irresolute. □

Theorem 2.5. *A function f:(X,τ_1,τ_2)→(Y,σ_1,σ_2) is (i,j)-completely fuzzy γ-irresolute if the graph function g:X→XXY is (i,j)-completely fuzzy γ-irresolute.*

Proof. Let $x{\in}X$ and V be a (i,j)-fuzzy γ-open set containing $f(x)$.Thus $X\text{X}V$ is a (i,j)-fuzzy γ-open set of $X\text{X}Y$ containing $g(x)$. Now $g^{-1}(X\text{X}V){=}\ f^{-1}(V)$ is a(i,j)-fuzzy regular open set containing x.This shows that f is(i,j)-completely fuzzy γ-irresolute.

The next theorem is established to study the composition of (i,j)-completely fuzzy γ-irresolute, (i,j)-fuzzy γ-irresolute and (i, j)-fuzzy γ- continuous functions . □

Theorem 2.6. *For a function f:(X,τ_1,τ_2)→(Y,σ_1,σ_2) and g:(Y,σ_1,σ_2)→(Z,η_1,η_2) the following statements hold true:*

i. *If f is (i,j)-completely fuzzy γ- irresolute and g is (i,j)-completely fuzzy γ-irresolute, then gof is (i,j)-completely fuzzy γ-irresolute.*

ii. *If f is (i,j)-completely fuzzy γ -irresolute and g is (i,j)-fuzzy γ- irresolute, then gof is (i,j)-completely fuzzy γ-irresolute.*

iii. *If f is (i,j-completely fuzzy γ-irresolute and g is (i, j)-fuzzy γ-continuous, then gof is (i,j)- completely fuzzy γ-irresolute.*

Proof. Proof is obvious. □

Definition 2.3. *A sub set B in a fuzzy bitopological space (X,τ_1,τ_2) is called (i,j)-fuzzy generalized closed set if for any τ_i-fuzzy open set U, τ_j-$Fcl(B)\subset U$ whenever $B\subset U$.*

Theorem 2.7. *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a map from X to Y and if $f^{-1}(A)$is both τ_i-fuzzy open set and (i,j)-fuzzy generalized closed set in X then f is a (i,j)-completely fuzzy γ-irresolute function.*

Proof. Let B be a (i,j)-fuzzy γ- open set in Y. Therefore from the hypothesis, we have $f^{-1}(B)$ is τ_i-fuzzy open set in X. Thus $f^{-1}(B)\subseteq$ i-int(j-cl($f^{-1}(B)$)). Again from the hypothesis we get i-int(j-cl($f^{-1}(B)$)$\subseteq f^{-1}(B)$.

Therefore from the above two relations we got
$f^{-1}(B)$=i-int(j-cl($f^{-1}(B)$)). Thus $f^{-1}(B)$ is a(i,j)-fuzzy regular open set in X.

Hence f is (i,j)-completely fuzzy γ -irresolute function. □

3. (i,j)-Fuzzy γ^*-Cosed Mapping and (i,j)-Fuzzy-γ-Normal Spaces

In this section we introduce and study the concepts of (i,j)-fuzzy γ^*-closed mapping and (i,j)-fuzzy γ-normal spaces. Also some of their properties are investigated.

Definition 3.1. *A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called (i,j)-fuzzy γ^*-closed mapping if the image of each (i,j)-fuzzy γ-closed set of X is (i,j)-fuzzy γ-closed set in Y.*

Theorem 3.1. *If a mapping $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is (i,j)-fuzzy γ^*-closed mapping then for each subset B of Y and each (i,j) -fuzzy γ-open set U of X containing $f^{-1}(B)$there exists a (i,j)-fuzzy γ -open set V containing B, such that $f^{-1}(V)\subset U$.*

Proof. Let f is (i,j)-fuzzy γ^*- closed mapping.Let $B\subset Y$ and U be any (i,j)-fuzzy γ-open set in X such that $f^{-1}(B)\subset U$. But since f is a (i,j)-fuzzy γ^*-closed mapping, so $(f(U^c))^c=V$ is (i,j)-fuzzy γ-open set in Y containing B such that $f^{-1}(V)\subset U$. □

Theorem 3.2. *Let(X,τ_1,τ_2) and (Y,σ_1,σ_2) be two fuzzy bitopological spaces.If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is bijective mapping then followings are equivalent:*

i. f is(i,j)-fuzzy γ^*-closed mapping and

ii. f^{-1} is (i,j)-fuzzy γ-irresolute mapping.

Proof. (i)→(ii) Let U be any (i,j)-fuzzy γ-closed set in X. Now since f is a (i,j)-fuzzy γ^*-closed mapping, then $(f^{-1})^{-1}(U)= f(U)$ is (i,j)-fuzzy γ-closed set in Y. Hence f^{-1} is (i,j)-fuzzy γ- irresolute mapping .

(ii)→(i) Let V be any (i,j)-fuzzy γ-closed set in X. Now since f^{-1} is (i,j)-fuzzy γ-irresolute mapping ,so$(f^{-1})^{-1}(V^c)$=$f(V^c)$ is (i,j)-fuzzy γ-open set in Y. Thus $f(V)$is (i,j)-fuzzy γ-closed set in Y. Hence f is (i, j)-fuzzy γ^*-closed mapping. □

Theorem 3.3. *Let f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ and g:$(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ be two mappings.If gof is (i,j)-fuzzy γ^*-closed mapping and f is (i,j)-fuzzy γ-irresolute surjection, then g is also (i,j)-fuzzy γ^*-closed map.*

Proof. Let A be any (i,j)-fuzzy γ- closed set in Y. Thus from the hypothesis, $f^{-1}(A)$ is (i,j)-fuzzy γ-closed set in X and $gof(f^{-1}(A))= g(A)$ is (i,j)-fuzzy γ-closed set in Z. Hence g is (i,j)-fuzzy γ^*- closed map. □

Theorem 3.4. *Let f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$be a mapping between two fuzzy bitopological spaces. Then followings are equivalent:*

i. f is (i,j)-fuzzy γ^*-closed mapping and

ii. (i,j)-Fγcl(f(A))⊂f((i,j)-Fγcl(A)),for every subset A of X.

Proof. (i)→(ii) $A\subset$(i,j)- Fγcl(A)
⇒ $f(A)\subset f$((i,j) -Fγcl(A))
⇒(i,j)-Fγcl(A))⊂(i,j)-Fγcl(f((i,j)-Fγcl(A)=f((i,j)-Fγcl(A))[by (i)].

(ii)→(i) Let B be a (i,j)-fuzzy γ- closed set in X.
Now by (ii)(i,j)-Fγcl($f(B)$)⊂f((i,j)-Fγcl(B))=$f(B)$.
⇒ (i,j)-Fγcl($f(B)$)⊂$f(B)$.
This shows that f is (i,j)-fuzzy γ^*-closed map. □

Definition 3.2. *A space (X,τ_1,τ_2) is said to be (i,j)-fuzzy normal if for each pair of disjoint τ_i-fuzzy closed set A and τ_j-fuzzy closed set B there exists two disjoints sets U and V such that U is τ_j -fuzzy open and V is τ_i-fuzzy open ,A⊂U and B⊂ V, where i≠j and i, j= 1 , 2.*

Example 3.1. *Let* $X=\{a,b\}$ *and consider the fuzzy sets on* X *are* $\overline{0},\overline{1},A=\{(a,0.8),(b,1)\}$, $B=\{(a,0),(b,0.7)\}$, $C=\{(a,1),(b,0.7)\}$ *and* $D=\{(a,0.3),(b,0)\}$. *Also let* $\tau_1=\{\overline{0},\overline{1},A,B\}$ *and* $\tau_2=\{\overline{0},\overline{1},C,D\}$ *be two fuzzy topologies on X. Here* $(X-A)$*and* $(X-C)$ *be two disjoint* τ_1*-fuzzy closed set and* τ_2*-fuzzy closed set in X which are contained in respectively two disjoint* τ_2*-fuzzy open set and* τ_1*-fuzzy open set B and D.*

Definition 3.3. *A space* (X,τ_1,τ_2) *is said to be (i,j)-fuzzy* γ*-normal if for any disjoint subsets* $F_1,F_2\in$ *(i,j)-*$F\gamma C(X)$*there exist disjoints subsets* $U,V\in$*(i,j)-*$F\gamma O(X)$ *such that* $F_1\subset U$ *and* $F_2\subset V$*, where* $i\neq j$ *and i, j= 1 , 2.*

Remark 3.1. *The next example shows that every* τ_1*-fuzzy open set may not be a (1,2)-fuzzy* γ*-open set. Also every* τ_2*-fuzzy open set may not be (2, 1)-fuzzy* γ*-open set.*

Example 3.2. *Let* $X=\{a,b\}$ *and consider the fuzzy sets on X are* $\overline{0},\overline{1},A=\{(a,0.2),(b,0.3)\},B=\{(a,0.9),(b,0.7)\}$. *Also let* $\tau_1=\{\overline{0},\overline{1},A\}$ *and* $\tau_2=\{\overline{0},\overline{1},B\}$ *be two fuzzy topologies on X.Here* τ_1*-fuzzy open set A is not a (1,2)-fuzzy* γ*-open set.Again* τ_2*-fuzzy open set B is not a (2,1)-fuzzy* γ*-open set.*

From this above example we can say that every (i,j)-fuzzy normal space may not be (i,j)-fuzzy γ-normal space , where $i\neq j$ and i, j =1, 2.

Question 3.1. *Is there an example of disjoint (i,j) -fuzzy* γ*-open sets showing that (i,j)-fuzzy* γ*-normality does not imply (i,j)-fuzzy normality.*

The next theorem is given for the condition under which (i,j)-fuzzy γ-normality is preserved.

Theorem 3.5. *If* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is (i,j)-fuzzy* γ*-irresolute and (i,j)-fuzzy* γ^**-closed surjection and X is (i, j)-fuzzy* γ*-normal then Y is (i,j)-fuzzy* γ*-normal.*

Proof. Let $F_1,F_2\in$(i,j)-FγC(Y)such that $F_1\cap F_2=\overline{0}$. Since f is a (i,j)-fuzzy γ-irresolute ,therefore $f^{-1}(F_1)\cap f^{-1}(F_2)=\overline{0}$ and $f^{-1}(F_1),f^{-1}(F_2)\in$(i,j)-F$\gamma$C($X$).Again from the hypothesis X is a (i,j)-fuzzy γ-normal so there exist disjoints subsets $U,V\in$(i,j)-FγO(X)such that $f^{-1}(F_1)\subset U$ and $f^{-1}(F_2)\subset V$. Again since f is a (i,j)-fuzzy γ^* -closed surjection, therefore $f\ (U^c)$and $f\ (V^c)\in$(i,j)-FγC(Y)and $f^{-1}(F_1)\subset U$ implies$F_1\subset G,F_2\subset H$ such that $G\cap H=\overline{0}$. □

4. Concluding Remarks

A (i,j)-fuzzy γ-irresolute function may not be a (i,j)-completely fuzzy γ irresolute function but in this paper we created a condition under which the fact is true.We have obtain some sufficient conditions which are useful for further study on (i, j)-completely fuzzy γ- irresolute function and related concepts.Also in this paper we have study the notion of (i,j)-fuzzy γ^*- closed mapping and shown that its inverses is a (i,j)-fuzzy γ- irresolute mapping.Finally it is found that (i,j)-fuzzy γ-normality is preserve under (i,j)-fuzzy γ^*- closed mapping.

References

Aygun, H., W. M. W. and Kudri, S. (1999). Completely induced l-fuzzy topological spaces. *Fuzzy Sets and Systems*, 103:513–523.

Bhaumik, R.N., M. A. and Pal, A. (1993). Fuzzy completely irresolute and fuzzy weakly irresolute functions. *Fuzzy Sets and Systems*, 59:79–85.

Chitharanjan, D. and Missier, P. (2012). On completely irresolute and weakly irresolute functions. *Kyungpook Math*, 52:99–107.

Ekici, E. and Jafari, S. (2008). On a weaker form of complete irresoluteness. *Bol. Soc. Paran. Mat.*, 26:81–87.

Kandil, A. (1989). Biproximaties and fuzzy bitopological spaces. *Simon Stevin.*, 63:45–66.

Kelly, J. (1963). Bitopological spaces. *Proc. London Math. Soc.*, 13:71–89.

Khedr, F. H. and Al-Saadi, H. S. (2005). Fuzzy pairwise almost strong pre continuity. *Kragujevac J. Math.*, 28:193–206.

Khedr, F. H. and Al-Saadi, H. S. (2012). Pairwise weakly and pairwise strongly irresolute functions. *J. of the Egyptian Mathematical Society.*, 20:7–13.

Mukherjee, A. and Bhattacharya, B. (2003). Pre -induced l- fuzzy supra topological spaces. *Indian J. Pure. Appl. Math.*, 28:1487–1493.

Mukherjee, M. N. (1985). On pairwise s-closed bitopological spaces. *Int.J. Math. Math.Sci.*, 8:729–745.

Naschie, M. (1998). On the uncertainty of cantorian geometry and the two-slit experiment. *Chaos, Solitons and Fractals*, 9:517–529.

Naschie, M. (2000). On the certification of heterotic strings, theory and theory. *Chaos, Solitons and Fractals*, pages 2397–2408.

Nour, T. M. (1995). A note on five separation axioms in bitopological spaces. *Indian J. Pure Appl. Math.*, 26:675–678.

Park, J. H., P. Y. B. and J., C. S. (1998). Fuzzy completely pre-irresolute and weakly completely pre-irresolute mappings. *Indian J. Pure Appl. Math.*, 97:115–121.

Rajesh, N. (2011). Two new types of irresolute functions via b-open sets. *Acta Universitatis Apulensis.*, 27:287–297.

Thakur, S. and Malviya, R. (1995). Pairwise fuzzy γ- irresolute mappings. *Mathematica Bohemica*, 3:273–280.

Tripathy, B. and Debnath, S. (2013). γ-open sets andγ-continuous mappings in fuzzy bitopological spaces. *Journal of Intelligent and Fuzzy Systems*, 24:631–635.

Tripathy, B. and Sharma, D. (2013). Pairwise strongly b-open and strongly b-closed functions in bitopological spaces. *Int. J. of Modern Math. Sci.*, 7.

Zadeh, L. A. (1965). Fuzzy sets. *Inform. and Control*, 8:338–353.

Zorlutuna, I. (2008). On strong forms of completely irresolute functions. *Chaos, Solitons and Fractals*, 38:970–979.